Nuclear Physics B299 (1988) 183–205
North-Holland, Amsterdam

COVARIANT UNCONSTRAINED SUPERFIELD ACTION FOR THE LINEARIZED $D = 10$ SUPER-YANG-MILLS THEORY

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Received 13 July 1987

A new type of $D = 10$ harmonic superspace with two generations of harmonics allows us to reduce the $D = 10$, $N = 1$ Brink-Schwarz (BS) superparticle to a system whose constraints are all first class, functionally independent and Lorentz-covariant. Given these properties, the covariant BFV-BRST quantization of the system is straightforward. By second quantizing this system, we circumvent the no-go theorem which forbids the existence of a covariant off-shell unconstrained superfield action for the linearized $D = 10$ super-Yang-Mills theory.

1. Introduction

Much progress was made recently in the Lorentz covariant first and second quantization of the Ramond-Neveu-Schwarz (RNS) spinning string [1, 2] (for a long list of references see the book [3]).

Since RNS has only first class constraints, the Batalin-Fradkin-Vilkovisky-Becchi-Rouet-Stora-Tyutin (BFV-BRST) approach [4, 5] applies quite directly.

In view of the importance of manifest space-time supersymmetry (anomaly cancellation, finiteness, vanishing cosmological constant etc.) for the superstrings [6, 3], it is of interest to obtain similar results also for the Green-Schwarz (GS) superstrings [7, 8].

The quantization of the GS superstring as well as the quantization of its point-limit – the Brink-Schwarz (BS) superparticle – [9, 10] presents, however, two major problems:

(i) It contains second class constraints which lead to highly complicated canonical Dirac brackets [8].

(ii) Both the second class constraints as well as the first class constraints – the local fermionic κ-symmetries – are functionally dependent when expressed covariantly. In the terminology of [11, 12], they form a reducible set.

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0550-3213/88/$03.50 © Elsevier Science Publishers B.V.
(North-Holland Physics Publishing Division)

In spite of intense efforts to overcome them, these problems prevented for years the consistent covariant quantization of the GS and BS systems.

In ref. [13] it was pointed out that for the GS and BS systems, the BFV procedure [11, 12] of treating correctly reducible constraints breaks Lorentz covariance if the level of reducibility is to remain finite*.

A way to solve this problem was proposed therein by pursuing the idea of introducing additional pure gauge-degrees of freedom – bosonic Lorentz-spinor harmonic variables – which permit a covariant irreducible formulation of the constraints.

The concept of harmonic superspace was first introduced for $D=4$ and in a different context in refs. [15]. Further, a covariant "light-cone" harmonic superspace in $D=10$ was proposed in ref. [16]. The latter, however, contains Lorentz-vector harmonics only, which are insufficient for a covariant and irreducible disentangling of the $D=10$ fermionic superparticle constraints.

Some unpleasant features in the formalism of [13] (obscure geometrical meaning of the spinor harmonics, BRST charge of rank 2) were overcome and an essential further progress was reported in ref. [17]:

(a) A significantly simpler and geometrically more meaningful construction of the $D=10$ harmonic superspace was presented. Now the bosonic harmonic coordinates carried Lorentz-spinor as well as Lorentz-vector indices. Also, the rank of the new Q_{BRST} was one.

(b) A covariant off-shell unconstrained superfield action for the linearized $D=10$ type II B supergravity was found.

(c) The mechanism of converting second-class constraints into first-class ones preserving all the way the physical content of the theory (i.e. without introducing unphysical degrees of freedom) was elucidated.

The procedure of ref. [17], however, exploited in an essential way the existence in the type II B theory of two Majorana-Weyl spinors of the same handedness. This was unfortunate because it excluded some very interesting cases.

In the present paper, we give a generalization of our treatment of the BS superparticle which applies equally well both to $N=1$ and $N=2$ (type II B as well as type II A) cases.

Using the BFV-BRST approach for the covariant second quantization of constrained systems [18], we also find an off-shell unconstrained superfield action for the linearized $D=10$, $N=1$ super-Yang-Mills (SYM) theory. In ref. [17] the analog result for the linearized $D=10$, $N=2$ supergravity was reported.

These results emphasize the strengths of the harmonic superspace techniques. Indeed, outside the harmonic superspace framework, the construction of the above

* The recently proposed formalism of ref. [14] for BFV quantization of the heterotic string with finite level of reduciblity breaks the Lorentz-invariance explicitly by introducing two *constant light-like vectors* which are *not* dynamical degrees of freedom.

covariant actions is probably impossible. (See refs. [19], [20] for the exact formulation of the respective no-go theorems.)

We use the harmonic variables introduced in ref. [17] to separate covariantly and irreducibly the first and second class constraints. We introduce a second generation of harmonic variables – the same as in ref. [21] – which allows us to reduce covariantly the $8N$ second class constraints to $4N$ first class constraints.

In essence we recognize half of the second class constraints as the gauge fixing conditions for the other half which are thereby recognized as generators of new gauge transformations. By renouncing the gauge fixing, these generators become first class constraints to which one can apply the BFV-BRST approach.

It is useful to consider the present paper in conjunction with ref. [21] where we quantized covariantly the Green-Schwarz superstring.

The harmonic structure is identical in the two papers.

However, in the superstring case it is impossible to repeat without further modifications the operations which permit us here to reduce the $8N$ second-class constraints to an equivalent set of $4N$ first class constraints. In ref. [21] we were obliged to proceed by *fixing covariantly* the κ- and reparametrization invariances.

Therefore, the present paper offers a glimpse to the kind of structure we would have obtained in the superstring case if we could go through with the quantization while keeping explicit all the gauge invariances.

The plan of the paper is as follows. In sect. 2, we present the general strategy. In sect. 3, the basic properties of the two generations of harmonic variables are recapitulated. (The reader might find useful to consult [17, 21] for technical details.) In sect. 4 we construct the harmonic form of the BS action. It contains only first class irreducible Lorentz-invariant constraints. In sect. 5 we perform in detail the covariant first quantization of the $N=1$ super-particle. The spectrum is identified as that of $D=10$, $N=1$ SYM. In sect 6 we discuss the BFV-BRST covariant second quantization. The first-quantized BRST charge of the $N=1$ BS superparticle is used to construct an off-shell covariant unconstrained superfield action of the linearized $D=10$, $N=1$ SYM.

2. General strategy

The standard form of the BS superparticle action in $D=10$ superspace (x^μ, θ_A)* written in the hamiltonian form reads:

$$S=\int \mathrm{d}\tau\left[p_\mu\,\partial_\tau x^\mu+\sum_A p_{\theta A}\,\partial_\tau\theta_A-H_{\mathrm{T}}\right], \tag{2.1}$$

$$H_{\mathrm{T}}=\lambda p^2+\sum_A \psi_A d_A\,. \tag{2.2}$$

* $A=1$ for $N=1$ and $A=1,2$ for $N=2$. For the spinor notations and conventions, see the appendix.

In eq. (2.1), θ_A are Majorana-Weyl (MW) spinors. For $N = 1$ we take $\theta = \theta_\alpha$ to be left handed. For $N = 2$ there are two cases:

(i) type II B – both $\theta_A = \theta_{A\alpha}$ have equal (left-handed) chiralities;

(ii) type II A – the two θ_A have opposite chiralities: $\theta_1 = \theta_{1\alpha}$, $\theta_2 = \theta_2^\alpha$ $(\equiv -C^{\alpha\dot{\beta}}\theta_{2\dot{\beta}})$ (here $C^{\alpha\dot{\beta}}$ denotes the $D = 10$ charge conjugation matrix, see the appendix).

The fermionic constraints

$$d_A = -ip_{\theta A} - \not{p}\theta_A\,, \tag{2.3}$$

$$\{d_A, d_B\}_{\mathrm{PB}} = 2i\delta^{AB}\not{p}^{\alpha\beta}\,, \tag{2.4}$$

form a mixture of $8N$ first class and $8N$ second class constraints on the constraint surface $p^2 = 0$.

Therefore, only half of ψ_A's are arbitrary Lagrange multipliers, the other half being determined from the consistency of the dynamics determined by H_{T} (2.2) with the whole set of constraints [22].

A serious drawback of (2.1) is that it is impossible to disentangle in a Lorentz-covariant and functionally independent way the first-class and the second-class parts of (2.3) [23].

In general it is always preferable, and sometimes essential, to work with first-class constraints only. (The appearance of inverses of operators in the Dirac brackets might interfere with a local formulation of the theory [3].) Therefore, instead of the initial mixed set of $8N$ first and $8N$ second class constraints d_A (2.3) we want to take a set of constraints containing just $12N$ Lorentz-covariant irreducible first class constraints. Of course we want the new system to describe the same physics as the original one.

We will achieve this task in two steps, each of them based on the use of a different type of harmonics.

The first step consists in separating the first class from the second class constraints and expressing each set covariantly, still, irreducibly. The harmonics that we use at this step are identical with the ones in ref. [17] and parameterize the coset $SO(1,9)/SO(8) \times SO(1,1)$. In fact the $8N$ first class constraints are represented with the help of these harmonics as N sets of SO(8) (s)-spinors whose components are all Lorentz-scalars. The same is true for the $8N$ second class constraints.

The second step is to recognize the set of $8N$ second class constraints as a set of $4N$ first class constraints together with their respective gauge fixings. This step is realized with the help of a second set of harmonics parameterizing the coset $SO(8)/SU(4) \times U(1)$ [21]. In fact, the $4N$ first class constraints are expressed with the help of these harmonics as N SO(6) (s)-spinors (i.e. 4 of SU(4)) whose components are SO(8) scalars. The $4N$ "gauge fixings" are expressed as SO(6) (c)-spinors (i.e. $\bar{4}$ of SU(4)). By renouncing these gauge fixings, one is left with only irreducible covariant first class constraints which can be treated à la BFV-BRST.

The equivalence between $2K$ real second class constraints and K holomorphic first class constraints is explained in great detail in the appendix C of ref. [17].

In the next section, we introduce the harmonic variables.

3. Harmonic variables

In ref. [17], we proposed a $D=10$ harmonic superspace whose bosonic harmonic coordinates consist of the following objects:

(a) two $D=10$ left-handed MW spinors $v_\alpha^{\pm\frac{1}{2}}$,

(b) eight ($a=1,\dots,8$), $D=10$ Lorentz-vectors u_μ^a.

These variables were defined to satisfy the constraints:

$$\begin{aligned}
\left[v_\alpha^{+\frac{1}{2}}(\sigma^\mu)^{\alpha\beta}v_\beta^{+\frac{1}{2}}\right]\left[v_\gamma^{-\frac{1}{2}}(\sigma_\mu)^{\gamma\delta}v_\delta^{-\frac{1}{2}}\right] &= -1,\\
\left[v_\alpha^{\pm\frac{1}{2}}(\sigma^\mu)^{\alpha\beta}v_\beta^{\pm\frac{1}{2}}\right]u_\mu^a &= 0,\\
u_\mu^a u^{b\mu} &= C^{ab}.
\end{aligned}\tag{3.1}$$

In the last line of (3.1), C^{ab} denotes the $D=8$ charge conjugation matrix.

Under the local rotations of the internal subgroup $\mathrm{H}=\mathrm{SO}(8)\times\mathrm{SO}(1,1)$, u_μ^a transform as SO(8) (s)-spinors whereas $v_\alpha^{\pm\frac{1}{2}}$ carry $\pm\frac{1}{2}$ charge under SO(1, 1).

Because of the remarkable $D=10$ Fierz identity (cf. e.g. ref. [3]):

$$(\sigma_\mu)^{\alpha\beta}(\sigma^\mu)^{\gamma\delta}+(\sigma_\mu)^{\beta\gamma}(\sigma^\mu)^{\alpha\delta}+(\sigma_\mu)^{\gamma\alpha}(\sigma^\mu)^{\beta\delta}=0,\tag{3.2}$$

the composite vectors

$$u_\mu^{\pm}=v_\alpha^{\pm\frac{1}{2}}(\sigma_\mu)^{\alpha\beta}v_\beta^{\pm\frac{1}{2}}\tag{3.3}$$

are identically light-like. Using the set of $u_\mu^{\pm}$ (3.3) together with u_μ^a from (3.1) one obtains a realization of the coset space $\mathrm{SO}(1,9)/\mathrm{SO}(8)\times\mathrm{SO}(1,1)$.

Henceforth, we shall use the shorthand notations:

$$\begin{aligned}
&u_\mu^{\pm} \qquad \text{as in (3.3)},\\
&\sigma^a\equiv\sigma^\mu u_\mu^a,\\
&\sigma^{\pm}\equiv\sigma^\mu\left(v^{\pm\frac{1}{2}}\sigma_\mu v^{\pm\frac{1}{2}}\right).
\end{aligned}\tag{3.4}$$

The second generation of harmonics [21] realizes the coset space $\mathrm{SO}(8)/\mathrm{SU}(4)\times\mathrm{U}(1)$ through the variables w_a^k, $\overline{w}_a^k$ subjected to the constraints:

$$C_{kl}\left(w_a^k\overline{w}_b^l+w_b^k\overline{w}_a^l\right)=C_{ab},\tag{3.5}$$

or, equivalently:

$$\begin{aligned}
w_a^k w^{la} &= \overline{w}_a^k\overline{w}^{la}=0,\\
w_a^k\overline{w}^{la} &= C^{kl}.
\end{aligned}$$

Here $C^{ki} = C^{ik}$ denotes the $D = 6$ charge conjugation matrix (cf. the appendix) (recall that locally SU(4) ≈ SO(6)).

There are two groups, H = SO(8) × SO(1,1) and K = SU(4) × U(1), which act as *internal* groups of local rotations on w_a^k, $\bar{w}_a^k$. The latter transform as (8,0) under H and as $(4, +\frac{1}{2}), (4, -\frac{1}{2})$ under K, respectively.

The following harmonic differential operators (which preserve the harmonic constraints (3.1) (3.5)) play an important role in the present approach:

$$D^{ab} = u_\mu^a \frac{\partial}{\partial u_{\mu b}} - u_\mu^b \frac{\partial}{\partial u_{\mu a}} + w^{ak} \frac{\partial}{\partial w_b^k} + \bar{w}^{ak} \frac{\partial}{\partial \bar{w}_b^k} - w^{bk} \frac{\partial}{\partial w_a^k} - \bar{w}^{bk} \frac{\partial}{\partial \bar{w}_a^k},$$

$$D^{-+} = \tfrac{1}{2} v_\alpha^{+\frac{1}{2}} \frac{\partial}{\partial v_\alpha^{+\frac{1}{2}}} - \tfrac{1}{2} v_\alpha^{-\frac{1}{2}} \frac{\partial}{\partial v_\alpha^{-\frac{1}{2}}}, \tag{3.6}$$

$$D^{+a} = u_\mu^+ \frac{\partial}{\partial u_{\mu a}} + \tfrac{1}{2} v^{-\frac{1}{2}} \sigma^+ \sigma^a \frac{\partial}{\partial v^{-\frac{1}{2}}},$$

$$E^{IJ} = w_a^k \left(\tfrac{1}{2} \rho^{IJ}\right)_k^{\ l} \frac{\partial}{\partial w_a^l} + \bar{w}_a^k \left(\tfrac{1}{2} \tilde{\rho}^{IJ}\right)_k^{\ l} \frac{\partial}{\partial \bar{w}_a^l},$$

$$E^{+-} = \tfrac{1}{2} \left(w_a^k \frac{\partial}{\partial w_a^k} - \bar{w}_a^k \frac{\partial}{\partial \bar{w}_a^k} \right),$$

$$E^{+I} = \frac{1}{\sqrt{2}} w_a^k \left(\rho^I\right)_k^{\ l} \frac{\partial}{\partial \bar{w}_a^l}, \tag{3.7}$$

where we have used the notations:

$$\rho^{IJ} = \tfrac{1}{2}\left(\rho^I \tilde{\rho}^J - \rho^J \tilde{\rho}^I \right),$$

$$\tilde{\rho}^{IJ} = \tfrac{1}{2}\left(\tilde{\rho}^I \rho^J - \tilde{\rho}^J \rho^I \right),$$

with ρ^I, $\tilde{\rho}^I$ $(I = 1, \ldots, 6)$ being the $D = 6$ σ-matrices (cf. appendix).

D^{ab}, D^{-+} and D^{+a} form a closed algebra. The first two of them are easily recognized as generators of SO(8) × SO(1,1) where the subgroup SO(8) is taken in the (s)-spinor representation, whereas D^{+a} are precisely the half of the coset generators corresponding to SO(1,9)/SO(8) × SO(1,1).

Similarly, E^{IJ} and E^{+-} are recognized as generators of SU(4) × U(1), whereas E^{+I} are the half of the coset generators corresponding to SO(8)/SU(4) × U(1).

The general harmonic superfield $\Phi = \Phi(x, \theta_A; u, v, w)$ is defined through its harmonic expansion. It is most appropriately expressed by first expanding Φ with respect to SO(8)/SU(4) × U(1) and then, expanding the coefficient fields with respect to SO(1,9)/SO(8) × SO(1,1).

We have explicitly:

$$\Phi(x, \theta_A; u, v, w) = \sum_{n=0}^{\infty} \frac{1}{n!} w^2_{a_1 b_1} \dots w^2_{a_n b_n} \Phi_n^{[a_1 b_1] \dots [a_n b_n]}(x, \theta_A; u, v), \tag{3.8}$$

where $w^2_{ab} \equiv w^k_{[a} \overline{w}_{b]k}$ is the only independent SU(4) × U(1)-invariant combination of w's (3.5) and the coefficient fields Φ_n in (3.8) are singlets with respect to the "small" group SU(4) × U(1) and traceless in any pair of SO(8) indices.

A slightly more general situation which will occur in sect. 5 below is when the harmonic superfield (3.8) processes an overall U(1)-charge h which is not carried by the w's. The way to express this property is to add a "non-orbital" part to the U(1) generator E^{+-} which does not act on the w's:

$$\hat{E}^{+-} = \tfrac{1}{2}\left(w_a^k \frac{\partial}{\partial w_a^k} - \overline{w}_a^k \frac{\partial}{\partial \overline{w}_a^k} \right) + \hat{h}, \tag{3.9}$$

$$\hat{h}\Phi = h\Phi. \tag{3.10}$$

Of course, $\hat{E}^{+-}$ (3.9) obeys the same commutation algebra with E^{IJ}, E^{+I} as E^{+-} does.

On their turn, the components in (3.8) can be expanded:

$$\begin{aligned} \Phi_n^{[a_1 b_1] \dots [a_n b_n]}(x, \theta_A; u, v) = & \sum_{r,s=0}^{\infty} \left[u^{c_1}_{\mu_1} \dots u^{c_r}_{\mu_r} \right]_{\text{singlet part in } (c_1 \dots c_r)} \\ & \times v^{+\frac{1}{2}}_{\alpha_1} \dots v^{+\frac{1}{2}}_{\alpha_s} v^{-\frac{1}{2}}_{\alpha_{s+1}} \dots v^{-\frac{1}{2}}_{\alpha_{2s}} \\ & \times \Phi_{n;\, rs}^{[a_1 b_1] \dots [a_n b_n] \mu_1 \dots \mu_r \alpha_1 \dots \alpha_{2s}}(x, \theta_A), \end{aligned} \tag{3.11}$$

where the coefficient fields $\Phi_{n;\,rs}$ in (3.11) do not carry indices of the "small" group SO(8) × SO(1,1) except for the external ones and are similarly subject to tracelessness conditions explained in ref. [17]. In fact the harmonic expansions (3.8), (3.11) already reflect the fact that:

$$E^{IJ}\Phi = E^{+-}\Phi = 0,$$

$$D^{ab}\Phi = D^{-+}\Phi = 0.$$

Now, as already demonstrated in refs. [17], [21], if Φ (3.8), (3.11) satisfies the harmonic equations

$$E^{IJ}\Phi = E^{-+}\Phi = E^{+I}\Phi = 0,$$

$$D^{ab}\Phi = D^{-+}\Phi = E^{+a}\Phi = 0,$$

then it has the properties:

$$\Phi_n^{[a_1 b_1]\dots[a_n b_n]}(x,\theta_A;u,v)=0 \qquad \text{for } n\neq 0;$$

$$\Phi_{0;\,rs}^{\mu_1\dots\mu_r\alpha_1\dots\alpha_{2s}}(x,\theta_A)=0 \qquad \text{for } (r,s)\neq(0,0),$$

$$\Phi_{0,00}(x,\theta_A) \qquad \text{arbitrary}, \tag{3.12}$$

i.e. Φ is in fact *independent* of u, v, w.

The same result (3.12) is obtained for harmonic superfields (3.8) carrying an overall U(1) charge (3.10).

The property (3.12) expresses the statement (on the first-quantized level) about the pure gauge nature of the harmonics. It insures the complete equivalence between the harmonic superparticle action of the next section and the standard BS action (2.1). In spite of their pure gauge nature, the harmonic degrees of freedom play an essential role in the subsequent construction, where we perform a suitable transformation mixing in a nontrivial way θ_A and u, v, w so that we can solve covariantly the irreducible covariantly disentangled set of superparticle constraints (cf. sects. 4 and 5).

4. The harmonic superparticle

Equipped with the harmonic formalism of the preceding section, we can now explicitly implement the program described in sect. 2: the Lorentz-covariant separation of the first class and second class parts from the constraints (2.3) of the action (2.1) and the transform of the second class constraints into first class ones.

The solution of the first task is provided by the decomposition of the 16-component $D=10$ Lorentz-spinors d_A (2.3) into direct sums of two internal SO(8) (s)-spinors each of which consist of 8 Lorentz-scalar components.

For $N=1$ and $N=2$ type II B superparticle the decomposition is:

$$d_A^\alpha=\frac{1}{p^+}\left(\sigma^b v^{+\frac12}\right)^\alpha d_{Ab}^{+\frac12}+\left(p^+\right)^{-1}\left(\not{p}\sigma^+\sigma^b v^{-\frac12}\right)^\alpha g_{Ab}^{+\frac12}, \tag{4.1}$$

where $p^+\equiv v^{+\frac12}\not{p}v^{+\frac12}$ or, by inverting (4.1):

$$d_A^{+\frac12 a}=\left(v^{+\frac12}\sigma^a\not{p}d_A\right), \tag{4.2}$$

$$g_A^{+\frac12 a}=\tfrac12\left(v^{-\frac12}\sigma^a\sigma^+ d_A\right). \tag{4.3}$$

For $N=2$ type II A superparticle the decomposition is:

$$d_1^{\alpha} = \text{same decomposition as in (4.1)},$$

$$d_{2\alpha} = \frac{1}{\sqrt{2}\,p^+}\left(\sigma^+\sigma^b v^{-\frac{1}{2}}\right)_{\alpha} d_{2b}^{+\frac{1}{2}} + \frac{\sqrt{2}}{p^+}\left(\not{p}\sigma^b v^{+\frac{1}{2}}\right)_{\alpha} g_{2b}^{+\frac{1}{2}},$$

$$d_2^{+\frac{1}{2}a} = \left(v^{-\frac{1}{2}}\sigma^a\sigma^+\not{p}d_2\right),$$

$$g_2^{+\frac{1}{2}a} = \sqrt{\tfrac{1}{2}}\left(v^{+\frac{1}{2}}\sigma^a d_2\right).$$

The canonical Poisson brackets are now rewritten as:

$$\left\{d_A^{+\frac{1}{2}a}, d_B^{+\frac{1}{2}b}\right\}_{\mathrm{PB}} = -2i\delta_{AB}C^{ab}p^+p^2, \tag{4.4}$$

$$\left\{g_A^{+\frac{1}{2}a}, g_B^{+\frac{1}{2}b}\right\}_{\mathrm{PB}} = i\delta_{AB}C^{ab}p^+,$$

$$\left\{d_A^{+\frac{1}{2}a}, g_B^{+\frac{1}{2}b}\right\}_{\mathrm{PB}} = 0. \tag{4.5}$$

The Poisson brackets imply that the $d_A^{+\frac{1}{2}a}$ are first class on the constraint surface $p^2=0$, while the $g_A^{+\frac{1}{2}a}$ are second class.

Now we will reduce the $8N$ second class constraints $g_A^{+\frac{1}{2}a}$ to $4N$ first class constraints.

We have to do it without breaking the harmonic SO(8) gauge invariance. This is achieved by using the second generation harmonic variables w_a^k described in the preceding section. We introduce the following linear combinations of the second class constraints:

$$g_A^{+\frac{1}{2}k} = w_a^k g_A^{+\frac{1}{2}a}, \tag{4.6}$$

$$\bar{g}_A^{+\frac{1}{2}\dot{k}} = \bar{w}_a^{\dot{k}} g_A^{+\frac{1}{2}a}. \tag{4.7}$$

The Poisson brackets of the new second class combinations are:

$$\left\{g_A^{+\frac{1}{2}k}, \bar{g}_A^{+\frac{1}{2}\dot{k}}\right\}_{\mathrm{PB}} = ip^+C^{k\dot{k}}\delta_{AB},$$

$$\left\{g_A^{+\frac{1}{2}k}, g_A^{+\frac{1}{2}k}\right\}_{\mathrm{PB}} = \left\{\bar{g}_A^{+\frac{1}{2}\dot{k}}, \bar{g}_A^{+\frac{1}{2}\dot{k}}\right\}_{\mathrm{PB}} = 0. \tag{4.8}$$

It is now clear that according to [17] one can discard the $\bar{g}_A^{+\frac{1}{2}\dot{k}}$ as mere gauge fixings and recognize $g_A^{+\frac{1}{2}k}$ as new first class constraints.

It is then immediate to write the hamiltonian as a sum of Lorentz-invariant irreducible first class constraints and to write the Q_{BRST} according to the standard formula (sect. 6).

Thus we arrive at the following form of the action for the $D=10$ BS superparticle:

$$S_{\text{harmonic}}=\int \mathrm{d}\tau\left[p_\mu\,\partial_\tau x^\mu+\sum_A p_{\theta A}\,\partial_\tau\theta_A+p^a_{u\mu}\,\partial_\tau u^\mu_a+p_v^{\mp\frac{1}{2}\alpha}\,\partial_\tau v_\alpha^{\pm\frac{1}{2}}\right.$$

$$\left.+p^a_{wk}\,\partial_\tau w^k_a+p^a_{\bar{w}\dot{k}}\,\partial_\tau\bar{w}^{\dot{k}}_a-H_{\text{harmonic}}\right],\tag{4.9}$$

$$H_{\text{harmonic}}=\lambda p^2+\sum_A\left(\lambda^{-\frac{1}{2}}_{A\dot{a}}d^{+\frac{1}{2}a}_A+\lambda^{-\frac{1}{2}}_{A\dot{k}}g^{+\frac{1}{2}k}_A\right)+\Lambda_{ab}d^{ab}+\Lambda^{+-}d^{-+}$$

$$+\Lambda^-_a d^{+a}+M_{IJ}e^{IJ}+M^{-+}e^{+-}+M^-_I e^{+I}.\tag{4.10}$$

The purely harmonic constraints d^{ab}, d^{+-}, d^{+a}, e^{IJ}, e^{+-}, e^{+I} in (4.10) denote the classical counterparts of D^{ab}, D^{-+}, D^{+a}, E^{IJ}, E^{+-}, E^{+I} (3.6), (3.7). λ, $\lambda_A^{-\frac{1}{2}a},\ldots,M^-_I$ are arbitrary Lagrange multipliers.

Because of the kinematical constraints (3.1) (3.5) on the variables u^a_μ, $v^{\pm\frac{1}{2}}_\alpha$, w^k_a, $\bar{w}^{\dot{k}}_a$ defining our harmonic superspace, their conjugate momenta are kinematically constrained too:

$$p_u^{\mu(a}u^{b)}_\mu=0,$$

$$p^a_{u_\mu}\left(v^{\pm\frac{1}{2}}\sigma^\mu v^{\pm\frac{1}{2}}\right)=0,$$

$$v^{+\frac{1}{2}}_\alpha p_v^{-\frac{1}{2}\alpha}+v^{-\frac{1}{2}}_\alpha p_v^{+\frac{1}{2}\alpha}=0,\tag{4.11}$$

$$p^a_{wk}\bar{w}^{\dot{l}}_a=0,$$

$$p^a_{\bar{w}k}w^l_a=0,$$

$$p^a_{\bar{w}\dot{k}}\bar{w}^{\dot{l}}_a=0,$$

$$p^a_{wk}w^l_a=0.\tag{4.12}$$

The constraints (3.1), (3.5) and (4.11), (4.12) may be equivalently regarded as a system of conjugated second class constraints and thus all subsequent Poisson-bracket relations are in fact Dirac-bracket relations on the surface defined by (3.1), (3.5), (4.11), (4.12).

5. Covariant first quantization

Proceeding to the quantization of S_{harmonic} (4.9) we shall discuss in detail the $N=1$ case. For $N=2$ type II B superparticle, the same results are recovered as in ref. [17] (where only the first generation of harmonics was used). For $N=2$ type II A superparticle the results are completely analogous to the latter case with the only difference occurring in the CPT properties of the spinorial coordinates.

According to (4.10), the covariant first-quantized Dirac-constrained equations for the $N=1$ harmonic superparticle read:

$$p^2\Phi = 0, \tag{5.1}$$

$$G^{+\frac{1}{2}k}\Phi \equiv w_a^k \tfrac{1}{2}\left(v^{-\frac{1}{2}}\sigma^a\sigma^+ D\right)\Phi = 0, \tag{5.2}$$

$$D^{+\frac{1}{2}a}\Phi \equiv v^{+\frac{1}{2}}\sigma^a \not{p} D\Phi = 0, \tag{5.3}$$

$$D^{ab}\Phi = 0, \qquad D^{-+}\Phi = 0, \qquad D^{+a}\Phi = 0, \tag{5.4}$$

$$E^{IJ}\Phi = 0, \qquad \left(\hat{E}^{+-} - h\right)\Phi = 0, \qquad E^{+I}\Phi = 0, \tag{5.5}$$

where now

$$D^\alpha = \frac{\partial}{\partial\theta_\alpha} - \not{p}^{\alpha\beta}\theta_\beta,$$

D^{ab}, D^{-+}, D^{+a}, E^{IJ}, $\hat{E}^{+-}$, E^{+I} are as in (3.6), (3.7), (3.9) and the superfield wave function $\Phi = \Phi(z)$ is taken in the p_μ momentum-representation space i.e. $z = (p_\mu, \theta;\, u, v, w)$, and may carry an overall U(1) charge h (3.10).

We want to analyse covariantly the physical content of the system characterized by (5.1)–(5.5).

Before doing this, let us briefly comment about the equivalence of the system (5.1)–(5.5) with the noncovariant light-cone formulation. Indeed, we can first solve eqs. (5.4), (5.5) in the original coordinate basis (central basis in the terminology of ref. [15]). The solution, given by (3.12), is then substituted into the remaining eqs. (5.1)–(5.3):

$$\begin{aligned} p^2\Phi_{0,00}(z) &= 0, \\ \tfrac{1}{2}w_a^k\left(v^{-\frac{1}{2}}\sigma^a\sigma^+ D\right)\Phi_{0,00}(z) &= 0, \\ v^{+\frac{1}{2}}\sigma^a\not{p}D\Phi_{0,00}(z) &= 0. \end{aligned} \tag{5.6}$$

Since in the last two eqs. (5.6) the harmonics are completely arbitrary and since, on the other hand, $\Phi_{0,00}(z)$ does not depend on them, we can actually choose the harmonics to point in fixed directions in Minkowski space consistent with the harmonic constraints (3.1), (3.5). In particular, choosing u_μ^a, $u_\mu^\pm$ $(\equiv v^{\pm\frac{1}{2}}\sigma_\mu v^{\pm\frac{1}{2}})$ to correspond to the transverse and light-cone space-time directions, respectively, and choosing w_a^k to correspond to the first half of the transverse dimensions, we arrive precisely to the [SU(4) × U(1)] light-cone description of $D=10$ SYM theory (ch. 11 of ref. [3]).

In order to solve *covariantly* the system (5.1)–(5.5) one has to perform on the θ's the same kind of covariant decomposition which we performed on the constraints d^α (4.1). We make first the following change of variables $\theta_\alpha \to \phi^{+\frac{1}{2}a}, \psi^{+\frac{1}{2}a}$:

$$\theta_\alpha = \frac{1}{2p^+}\left(\sigma^+\sigma^b v^{-\frac{1}{2}}\right)\phi_b^{+\frac{1}{2}} + \frac{1}{p^+}\left(\not{p}\sigma^b v^{+\frac{1}{2}}\right)\psi_b^{+\frac{1}{2}},$$

or, inversely:

$$\phi^{+\frac{1}{2}a} = v^{-\frac{1}{2}}\sigma^a\sigma^+\not{p}\theta\,,$$

$$\psi^{+\frac{1}{2}a} = v^{+\frac{1}{2}}\sigma^a\theta\,. \tag{5.7}$$

We further decompose $\phi^{+\frac{1}{2}a}$ (5.7) as follows:

$$\phi^{+\frac{1}{2}k} = w^k_a\phi^{+\frac{1}{2}a}\,, \qquad \bar{\phi}^{+\frac{1}{2}\dot{k}} = \bar{w}^{\dot{k}}_a\phi^{+\frac{1}{2}a}\,. \tag{5.8}$$

Now we can use a new set of canonical coordinates:

$$\left(p_\mu, \phi^{+\frac{1}{2}k}, \bar{\phi}^{+\frac{1}{2}\dot{k}}, \psi^{+\frac{1}{2}a}, u, v, w\right) \tag{5.9}$$

instead of the old one $(x^\mu, \theta_\alpha, u, v, w)$*.

In terms of (5.9) the (quantized) fermionic constraints take the particularly simple form:

$$G^{+\frac{1}{2}k} = p^+\frac{\partial}{\partial\bar{\phi}_k^{+\frac{1}{2}}} - \tfrac{1}{2}\phi^{+\frac{1}{2}k}\,, \tag{5.10}$$

$$D^{+\frac{1}{2}a} = p^+\frac{\partial}{\partial\psi_a^{+\frac{1}{2}}} + p^2\psi^{+\frac{1}{2}a} \tag{5.11}$$

Accordingly, the harmonic constraints (3.6), (3.7) take the following form in the new coordinate basis:

$$\tilde{D}^{ab} = D^{ab} + \psi^{+\frac{1}{2}a}\frac{\partial}{\partial\psi_b^{+\frac{1}{2}}} - \psi^{+\frac{1}{2}b}\frac{\partial}{\partial\psi_a^{+\frac{1}{2}}}\,,$$

$$\tilde{D}^{-+} = D^{-+} + \tfrac{1}{2}\left[\psi^{+\frac{1}{2}a}\frac{\partial}{\partial\psi_a^{+\frac{1}{2}}} + \phi^{+\frac{1}{2}k}\frac{\partial}{\partial\phi^{+\frac{1}{2}k}} + \bar{\phi}^{+\frac{1}{2}\dot{k}}\frac{\partial}{\partial\bar{\phi}^{+\frac{1}{2}\dot{k}}}\right],$$

$$\tilde{D}^{+a} = D^{+a}, \tag{5.12}$$

$$\tilde{E}^{IJ} = E^{IJ} + \tfrac{1}{2}\left[\phi^{+\frac{1}{2}k}\left(\rho^{IJ}\right)_k^{\ l}\frac{\partial}{\partial\phi^{+\frac{1}{2}l}} + \bar{\phi}^{+\frac{1}{2}\dot{k}}\left(\tilde{\rho}^{IJ}\right)_{\dot{k}}^{\ \dot{l}}\frac{\partial}{\partial\bar{\phi}^{+\frac{1}{2}\dot{l}}}\right],$$

$$\tilde{E}^{+-} = E^{+-} + \tfrac{1}{2}\left[\phi^{+\frac{1}{2}k}\frac{\partial}{\partial\phi^{+\frac{1}{2}k}} - \bar{\phi}^{+\frac{1}{2}\dot{k}}\frac{\partial}{\partial\bar{\phi}^{+\frac{1}{2}\dot{k}}}\right],$$

$$\tilde{E}^{+I} = E^{+I} + \phi^{+\frac{1}{2}k}\left(\rho^I\right)_{k\dot{l}}\left[\frac{\partial}{\partial\bar{\phi}_{\dot{l}}^{+\frac{1}{2}}} - \frac{1}{p^+}\bar{\phi}^{+\frac{1}{2}\dot{l}}\right]. \tag{5.13}$$

* We shall not need the complicated explicit formulas expressing the new canonical momenta conjugated to (5.9) as functions of the old coordinates and momenta.

With (5.10)–(5.13), the Lorentz-covariant solution of the system (5.1)–(5.5) becomes completely straightforward. We obtain:

$$\Phi\left(p, \phi^{+\frac{1}{2}k}, \bar{\phi}^{+\frac{1}{2}k}, \psi^{+\frac{1}{2}a}, u, v, w\right)$$
$$= \exp\left(\frac{1}{2p^{+}} \bar{\phi}_{k}^{+\frac{1}{2}} \phi^{+\frac{1}{2}k}\right) \hat{\Phi}\left(p, \phi^{+\frac{1}{2}k}, u, v, w\right)$$
$$= \exp\left(\frac{1}{2p^{+}} \bar{\phi}_{k}^{+\frac{1}{2}} \phi^{+\frac{1}{2}k}\right) \sum_{n=0}^{4} \frac{1}{n!} \phi^{+\frac{1}{2}k_1} \ldots \phi^{+\frac{1}{2}k_n} \hat{\Phi}_{k_1 \ldots k_n}^{(-n/2)}(p; u, v, w), \quad (5.14)$$

$$p^2 \hat{\Phi}_{k_1 \ldots k_n}^{(-n/2)}(p; u, v, w) = 0,$$
$$D^{+a} \hat{\Phi}_{k_1 \ldots k_n}^{-(n/2)}(p; u, v, w) = 0,$$
$$E^{+I} \hat{\Phi}_{k_1 \ldots k_n}^{-(n/2)}(p; u, v, w) = 0. \quad (5.15)$$

Moreover, we can impose the following covariant *off-shell* reality condition on Φ, expressing its *CPT* self-conjugacy property [24]:

$$\left[\Phi\left(-p, \phi'^{+\frac{1}{2}k}, \phi^{+\frac{1}{2}k}, \psi^{+\frac{1}{2}a}, u, v, w\right)\right]^{*}$$
$$= (p^{+})^{2} \int \mathrm{d}^{4}\theta^{+\frac{1}{2}k} \exp\left\{i \frac{\bar{\phi}'^{+\frac{1}{2}}_{k} \theta^{+\frac{1}{2}k}}{p^{+}}\right\} \Phi\left(p, \theta^{+\frac{1}{2}k}, \bar{\phi}^{+\frac{1}{2}k}, \psi^{+\frac{1}{2}a}, u, v, w\right). \quad (5.16)$$

As a consequence of (5.16) we get that Φ carries an overall U(1) charge $h = 1$ (cf. eq. (3.10)).

Comparing (5.14)–(5.16) with the corresponding formulae in the non-covariant light-cone quantization of $N = 1$ BS superparticles (eqs. (11.7.25)–(11.7.27) of ref. [3]) we see that the above equations constitute a Lorentz-covariant on-shell superfield description of $D = 10$ linearized SYM theory. The off-shell (i.e. action-principle) description will be given in the next section.

6. BFV-BRST quantization

According to the general theory [4], the BRST charge Q_{BRST} may contain higher order ghost terms if the canonical PB relations among the first-class constraints involve nontrivial first-order structure functions (i.e. structure "constants" of the algebra of constraints which depend on the canonical variables).

The latter situation indeed arises in one of the PB relations for the harmonic superparticle constraints (4.4)–(4.5).

However, one can check straightforwardly, e.g. by using the explicit formulas of ref. [4], that in the present case the second order structure functions and, therefore, all higher structure functions identically vanish.

Thus, Q_{BRST} of the $N=1$ harmonic superparticle (4.9) is first rank:

$$Q_{\mathrm{BRST}} = Q_0 + Q^{(\mathrm{I})}_{\mathrm{harmonic}} + Q^{(\mathrm{II})}_{\mathrm{harmonic}}\,, \tag{6.1}$$

$$\begin{aligned} Q_0 = {} & \alpha^{-1}\left[cp^2 + i\left(c\frac{\partial}{\partial c} - \frac{1}{2}\right)\frac{\partial}{\partial \tilde{c}}\right] + i\frac{\partial}{\partial \alpha}\frac{\partial}{\partial \tilde{c}} + \bar{\chi}_k^{-\frac{1}{2}} G^{+\frac{1}{2}k} + \chi_a^{-\frac{1}{2}} D^{+\frac{1}{2}a} \\ & - \chi_a^{-\frac{1}{2}}\chi^{-\frac{1}{2}a} p^+ \alpha \frac{\partial}{\partial c} - \frac{\partial}{\partial \bar{\psi}_k^{-\frac{1}{2}}}\frac{\partial}{\partial \tilde{\chi}^{+\frac{1}{2}k}} - \frac{\partial}{\partial \psi_a^{-\frac{1}{2}}}\frac{\partial}{\partial \tilde{\chi}^{+\frac{1}{2}a}} \end{aligned} \tag{6.2}$$

where $\alpha \equiv \sqrt{2\lambda}$,

$$\begin{aligned} Q^{(\mathrm{I})}_{\mathrm{harmonic}} = {} & i\eta_{ab}\left[D^{ab} + \chi^{-\frac{1}{2}a}\frac{\partial}{\partial \chi_b^{-\frac{1}{2}}} - \chi^{-\frac{1}{2}b}\frac{\partial}{\partial \chi_a^{-\frac{1}{2}}} + \eta^{-a}\frac{\partial}{\partial \eta_b^-} - \eta^{-b}\frac{\partial}{\partial \eta_a^-}\right. \\ & \left. + \eta_d^a\frac{\partial}{\partial \eta_{bd}} - \eta_d^b\frac{\partial}{\partial \eta_{ad}}\right] \\ & + i\eta\left[D^{-+} - \tfrac{1}{2}\chi_a^{-\frac{1}{2}}\frac{\partial}{\partial \chi_a^{-\frac{1}{2}}} - \tfrac{1}{2}\bar{\chi}_k^{-\frac{1}{2}}\frac{\partial}{\partial \bar{\chi}_k^{-\frac{1}{2}}} - \eta_a^-\frac{\partial}{\partial \eta_a^-}\right] \\ & + i\eta_a^- D^{+a} + i\frac{\partial}{\partial \Lambda_{ab}}\frac{\partial}{\partial \tilde{\eta}^{ab}} + i\frac{\partial}{\partial \Lambda^{+-}}\frac{\partial}{\partial \tilde{\eta}} + i\frac{\partial}{\partial \Lambda^{-a}}\frac{\partial}{\partial \tilde{\eta}_a^+}\,, \end{aligned} \tag{6.3}$$

$$\begin{aligned} Q^{(\mathrm{II})}_{\mathrm{harmonic}} = {} & i\xi^{IJ}\left[E^{IJ} - \tfrac{1}{2}\bar{\chi}_k^{-\frac{1}{2}}(\tilde{\rho}^{IJ})_l^k\frac{\partial}{\partial \bar{\chi}_l^{-\frac{1}{2}}} + \xi^{-I}\frac{\partial}{\partial \xi^{-J}} - \xi^{-J}\frac{\partial}{\partial \xi^{-I}}\right. \\ & \left. + \xi^{IK}\frac{\partial}{\partial \xi^{JK}} - \xi^{JK}\frac{\partial}{\partial \xi^{IK}}\right] + i\xi\left[\hat{E}^{+-} - h - \tfrac{1}{2}\bar{\chi}_k^{-\frac{1}{2}}\frac{\partial}{\partial \bar{\chi}_k^{-\frac{1}{2}}} - \xi^{-I}\frac{\partial}{\partial \xi^{-I}}\right] \\ & + i\xi^{-I}E^{+I} + i\frac{\partial}{\partial M^{IJ}}\frac{\partial}{\partial \tilde{\xi}^{IJ}} + i\frac{\partial}{\partial M^{-+}}\frac{\partial}{\partial \tilde{\xi}} + i\frac{\partial}{\partial M^{-I}}\frac{\partial}{\partial \tilde{\xi}^{I}}\,. \end{aligned} \tag{6.4}$$

The variables appearing in the above expression of the BRST charge are organized as follows:

$$\begin{bmatrix} \textit{Lagrange multiplier} & \textit{ghost} & \textit{antighost} & \textit{of the constraint} \\ \alpha & c & \tilde{c} & p^2 \\ \bar{\psi}_k^{-\frac{1}{2}} & \bar{\chi}_k^{-\frac{1}{2}} & \tilde{\chi}^{+\frac{1}{2}k} & G^{+\frac{1}{2}k} \\ \psi_a^{-\frac{1}{2}} & \chi_a^{-\frac{1}{2}} & \tilde{\chi}_a^{+\frac{1}{2}} & D^{+\frac{1}{2}a} \\ \Lambda_{ab} & \eta_{ab} & \tilde{\eta}_{ab} & D^{ab} \\ \Lambda^{+-} & \eta & \tilde{\eta} & D^{-+} \\ \Lambda^{-a} & \eta^{-a} & \tilde{\eta}^{+a} & D^{+a} \\ M^{IJ} & \xi^{IJ} & \tilde{\xi}^{IJ} & E^{IJ} \\ M^{-+} & \xi & \tilde{\xi} & \hat{E}^{+-}-h \\ M^{-I} & \xi^{-I} & \tilde{\xi}^{+I} & E^{+I} \end{bmatrix}$$

It will be useful in the following to give the common name ζ to all these variables:

$$\zeta = \Big(\alpha, c, \tilde{c};\ \tilde{\psi}_k^{-\frac{1}{2}}, \tilde{\chi}_k^{-\frac{1}{2}}, \tilde{\chi}^{+\frac{1}{2}k};\ \psi_a^{-\frac{1}{2}}, \chi_a^{-\frac{1}{2}}, \tilde{\chi}_a^{+\frac{1}{2}};$$

$$\Lambda_{ab}, \eta_{ab}, \tilde{\eta}_{ab};\ \Lambda^{+-}, \eta, \tilde{\eta};\ \Lambda^{-a}, \eta^{-a}, \tilde{\eta}^{+a};\ M^{IJ}, \xi^{IJ}, \tilde{\xi}^{IJ};\ M^{-+}, \xi, \tilde{\xi};\ M^{-I}, \xi^{-I}, \tilde{\xi}^{+I}\Big). \tag{6.5}$$

Recall that from (5.16) the overall U(1) charge h must equal one.

Starting with the Q_{BRST} (6.1) we are now able to write down a covariant unconstrained superfield action for the linearized $D=10$, $N=1$ SYM theory along the lines of the Neveu-West approach [18].

Choosing a BFV gauge function $\Psi = \partial/\partial c$, the first quantized BFV hamiltonian [4]:

$$H_{\mathrm{BFV}} = \left\{ Q_{\mathrm{BRST}}, \frac{\partial}{\partial c} \right\} = \alpha^{-1}\left[p^2 + i\frac{\partial}{\partial c}\frac{\partial}{\partial \tilde{c}} \right]$$

has the same form as the one of the ordinary bosonic particle.

Accordingly, we find the following second-quantized BRST action:

$$S_{\mathrm{BRST}} = \int \mathrm{d}\tau\, \mathrm{d}z\, \mathrm{d}\zeta\, r^2(u, v, w)\big[\hat{K}p^+\Phi(\tau, z, \zeta)\big]$$

$$\times\left[i\partial_\tau - \frac{1}{\alpha}\left(p^2 + i\frac{\partial}{\partial c}\frac{\partial}{\partial \tilde{c}} \right)\right]\big[p^+\Phi(\tau, z, \zeta)\big]. \tag{6.6}$$

The operator $\hat{K}$ in (6.6) acts on the ζ-coordinates (6.5) only by changing the signs of all the Lagrange multipliers and of the bosonic ghosts $\lambda_a^{-\frac{1}{2}}$:

$$\hat{K}\Phi(\tau, z, \zeta) = \Phi\Big(\tau, z; -\alpha, \dots; -\bar{\psi}_k^{-\frac{1}{2}}, \dots; -\psi_a^{-\frac{1}{2}}, -\chi_a^{-\frac{1}{2}}, \dots; -\Lambda_{ab}, \dots; -\Lambda^{+-}, \dots;$$
$$-\Lambda^{-a}, \dots; \dots; -M^{IJ}, \dots; -M^{+-}, \dots; -M^{-I}, \dots\Big). \qquad (6.7)$$

The two factors p^+ (recall that $p^+ \equiv v^{+\frac{1}{2}} \not{p} v^{+\frac{1}{2}}$ is Lorentz invariant because the superscript + is internal) in (6.6) are needed to compensate for the SO(1,1) charge (-2) of the measure $\mathrm{d}\zeta$.

Similarly, the factor r^2 carrying U(1) charge two:

$$r^2(u, v, w) = \varepsilon_{k_1 \dots k_4} w_{a_1}^{k_1} \dots w_{a_4}^{k_4} \big(v^{+\frac{1}{2}} \sigma^{[a_1} \sigma^{a_2} \sigma^{a_3} \sigma^{a_4]} \sigma^- v^{+\frac{1}{2}}\big) \qquad (6.8)$$

is introduced in (6.6) to compensate for the U(1) charges (-4) of the measure $\mathrm{d}\zeta$ and $(+2)$ of the fields Φ. r^2 (6.8) possesses the following important properties:

$$D^{ab} r^2 = D^{-+} r^2 = D^{+a} r^2 = 0\,,$$
$$E^{IJ} r^2 = E^{+I} r^2 = 0\,, \qquad E^{+-} r^2 = 2r^2\,. \qquad (6.9)$$

The action (6.6) is invariant under the (second quantized) BRST transformation:

$$\delta_{\mathrm{BRST}} \Phi(\tau, z, \zeta) = \Lambda Q_{\mathrm{BRST}} \Phi(\tau, z, \zeta) \qquad (6.10)$$

due to the operator identity:

$$\hat{K} Q_{\mathrm{BRST}} = -(Q_{\mathrm{BRST}})^{\mathrm{T}} \hat{K} \qquad (6.11)$$

and because of the invariance properties of $r^2(u, v, w)$ (6.9).

The superscript "T" in (6.11) means operator transposition and Λ in (6.10) is a hermitian and anticommuting global parameter.

We impose the following covariant off-shell reality condition:

$$\hat{\Phi}^*(\tau, z, \zeta) = \hat{F} \hat{K} \Phi(\tau, z, \zeta)\,. \qquad (6.12)$$

$\hat{F}$ denotes the operator of grassmanian Fourier transform of exactly the same form as in (5.16) (i.e. $\hat{F}$ acts only on the original harmonic superspace coordinates $z = (p_\mu, \theta_\alpha, u, v, w)$).

Let us note that by construction:

$$\big[Q_{\mathrm{BRST}}, \hat{F}\big] = 0\,. \qquad (6.13)$$

Eqs. (6.11) and (6.13) insure the consistency of the reality condition (6.12) with the BRST transformation law (6.10).

One can now further simplify our action by taking the Fourier transform with respect to those variables which change sign under the action of $\hat{K}$ (6.7):

$$\Phi\Big(\tau, z; \alpha, \ldots; \bar{\psi}_k^{-\frac{1}{2}}, \ldots \psi_a^{-\frac{1}{2}}, \chi_a^{-\frac{1}{2}} \ldots; \Lambda_{ab}, \ldots; \Lambda^{+-}, \ldots; \Lambda^{-a}, \ldots; \ldots;$$
$$M^{IJ}, \ldots; M^{-+}, \ldots; M^{-I}, \ldots\Big)$$
$$= [r(u,v,w)]^{-4}(p^+)^{-10}\sqrt{\frac{\alpha}{2\pi}}$$
$$\times \int \mathrm{d}y\, \mathrm{d}^8\omega^{-\frac{1}{2}a}\, \mathrm{d}^8\rho^{-\frac{1}{2}a}\, \mathrm{d}^4\theta^{-\frac{1}{2}k}\, \mathrm{d}Y_{ab} \mathrm{d}Y^{-+}\, \mathrm{d}Y^{+a} \mathrm{d}Z^{IJ} \mathrm{d}Z^{+-}\, \mathrm{d}Z^{+I}$$
$$\times \exp\Big\{ i\alpha y + p^+ \bar{\psi}_k^{-\frac{1}{2}} \theta^{-\frac{1}{2}k} + p^+ \psi_a^{-\frac{1}{2}} \omega^{-\frac{1}{2}a} + ip^+ \chi_a^{-\frac{1}{2}} \rho^{-\frac{1}{2}a} + i\Lambda^{ab} Y_{ab}$$
$$+ i\Lambda^{+-} Y^{-+} + i\Lambda_a^- Y^{+a} + iM^{IJ} Z^{IJ} + iM^{-+} Z^{+-} + iM^{-I} Z^{+I} \Big\}$$
$$\times \tilde{\Phi}\Big(\tau, z; y, \ldots, \theta^{-\frac{1}{2}k}, \ldots, \omega_a^{-\frac{1}{2}}, \rho_a^{-\frac{1}{2}}, \ldots,$$
$$Y_{ab}, \ldots, Y^{-+}, \ldots, Y^{+a}, \ldots Z^{IJ}, \ldots, Z^{+-}, \ldots, Z^{+I}, \ldots\Big).$$

Then the action (6.6) acquires the form:

$$S_{\mathrm{BRST}} = \int \mathrm{d}\tau \mathrm{d}z \mathrm{d}\tilde{\zeta} [r(u,v,w)]^8 \Big[(p^+)^{-2} \tilde{\Phi}(\tau, z, \tilde{\zeta})\Big]$$
$$\times \left[-\frac{\partial}{\partial y}\frac{\partial}{\partial \tau} - p^2 - i\frac{\partial}{\partial c}\frac{\partial}{\partial \tilde{c}} \right] \Big[(p^+)^{-2} \tilde{\Phi}(\tau, z, \tilde{\zeta})\Big], \tag{6.14}$$

$$\tilde{\zeta} \equiv \Big(y, c, \tilde{c}; \theta^{-\frac{1}{2}k}, \bar{\chi}_k^{-\frac{1}{2}}, \tilde{\chi}^{+\frac{1}{2}k}; \omega_a^{-\frac{1}{2}}, \rho_a^{-\frac{1}{2}}, \tilde{\chi}_a^{+\frac{1}{2}}; Y_{ab}, \eta_{ab}, \tilde{\eta}_{ab}; Y^{-+}, \eta, \tilde{\eta};$$
$$Y^{+a}, \eta^{-a}, \tilde{\eta}^{+a}; Z^{IJ}, \xi^{IJ}, \tilde{\xi}^{IJ}; Z^{+-}, \xi, \tilde{\xi}; Z^{-I}, \xi^{-I}, \tilde{\xi}^{+I} \Big) \tag{6.15}$$

and the reality condition (6.12) becomes:

$$\tilde{\Phi}^*(\tau, z, \tilde{\zeta}) = \hat{F} \tilde{\Phi}(\tau, z, \tilde{\zeta}). \tag{6.16}$$

The construction of a covariant unconstrained superfield action for the linearized $D = 10$ SYM theory and the analog result for the linearized $D = 10$, $N = 2$ supergravity [17] circumvent the existing no-go theorems of refs. [19, 20]. The loophole allowing us to avoid these no-go theorems is hidden in the fact that the "ghost-haunted" harmonic superfield $\Phi(\tau, z, \zeta)$, while describing on-shell a finite number of degrees of freedom, contains off-shell an infinite number of gauge and auxiliary superfields.

In complete analogy with ref. [17], the BRST action (6.14) possesses Parisi-Sourlas [25, 26] symmetries under OSp(1,1|2) rotations [18, 27, 28] in the subspaces parametrized by the following $\tilde{\zeta}$ coordinates (6.15), respectively:

$$
\begin{aligned}
&(\tau, y; c, \tilde{c}), \\
&\left(\tilde{\chi}_a^{+\frac{1}{2}}, \rho_a^{-\frac{1}{2}}; \psi_a^{+\frac{1}{2}}, \omega_a^{-\frac{1}{2}}\right), \qquad \text{for each } a, \\
&\left(\tilde{\chi}^{+\frac{1}{2}k}, \bar{\chi}_k^{-\frac{1}{2}}; \bar{\phi}_k^{+\frac{1}{2}}, \theta^{-\frac{1}{2}k}\right), \qquad \text{for each } k.
\end{aligned} \tag{6.17}
$$

In eq. (6.17), $\psi_a^{+\frac{1}{2}}, \bar{\phi}_k^{+\frac{1}{2}}$ are the same as in (5.7), (5.8). Thus, after Parisi-Sourlas dimensional reduction [25, 26, 28, 29] we get the reduced BRST action:

$$
\begin{aligned}
S_{\mathrm{BRST}}^{(\mathrm{red})} &= \int \mathrm{d}^{10}p\, \mathrm{d}^4\phi^{+\frac{1}{2}k}\, \mathrm{d}u\, \mathrm{d}v\, \mathrm{d}w\, \mathrm{d}\tilde{\zeta}^{(\mathrm{red})} \left[r(u, v, w)\right]^6 \\
&\quad \times \left[(p^+)^{-3}\tilde{\Phi}^{(\mathrm{red})}\right] p^2 \left[(p^+)^{-3}\tilde{\Phi}^{(\mathrm{red})}\right], \\
\tilde{\zeta}^{(\mathrm{red})} &\equiv \left(Y_{ab}, \eta_{ab}, \tilde{\eta}_{ab}; Y^{-+}, \eta, \tilde{\eta}; Y_a^+, \eta_a^-, \tilde{\eta}_a^+;\right. \\
&\quad \left. Z^{IJ}, \xi^{IJ}, \tilde{\xi}^{IJ}; Z^{+-}, \xi, \tilde{\xi}; Z^{+I}, \xi^{-I}, \tilde{\xi}^{+I}\right).
\end{aligned} \tag{6.18}
$$

The action (6.18) is invariant under the reduced (second-quantized) BRST transformation:

$$
\begin{aligned}
&\delta_{\mathrm{BRST}} \tilde{\Phi}^{(\mathrm{red})} = \Lambda Q_{\mathrm{BRST}}^{(\mathrm{red})} \tilde{\Phi}^{(\mathrm{red})}, \\
&\tilde{Q}_{\mathrm{BRST}}^{(\mathrm{red})} = i\eta_{ab}\left(D^{ab} + \eta_d^a \frac{\partial}{\partial \eta_{bd}} - \eta_d^b \frac{\partial}{\partial \eta_{ad}} + \eta^{-a}\frac{\partial}{\partial \eta_b^-} - \eta^{-b}\frac{\partial}{\partial \eta_a^-}\right) \\
&\quad + i\eta\left(D^{-+} + \tfrac{1}{2}\phi^{+\frac{1}{2}k}\frac{\partial}{\partial \phi^{+\frac{1}{2}k}} - \eta^{-a}\frac{\partial}{\partial \eta^{-a}}\right) + i\eta_a^- D^{+a} \\
&\quad \times i\xi^{IJ}\left(E^{IJ} + \xi^{IK}\frac{\partial}{\partial \xi^{JK}} - \xi^{JK}\frac{\partial}{\partial \xi^{IK}} + \tfrac{1}{2}\phi^{+\frac{1}{2}k}(\rho^{IJ})_k^l \frac{\partial}{\partial \phi^{+\frac{1}{2}l}} + \xi^{-I}\frac{\partial}{\partial \xi^{-J}} - \xi^{-J}\frac{\partial}{\partial \xi^{-I}}\right) \\
&\quad + i\xi\left(E^{+-} + \tfrac{1}{2}\phi^{+\frac{1}{2}k}\frac{\partial}{\partial \phi^{+\frac{1}{2}k}} - \xi^{-I}\frac{\partial}{\partial \xi^{-I}}\right) + i\xi^{-I}E^{+I} - Y^{ab}\frac{\partial}{\partial \tilde{\eta}^{ab}} - Y^{-+}\frac{\partial}{\partial \tilde{\eta}} \\
&\quad - Y^{+a}\frac{\partial}{\partial \tilde{\eta}^{+a}} - Z^{IJ}\frac{\partial}{\partial \tilde{\xi}^{IJ}} - Z^{+-}\frac{\partial}{\partial \tilde{\xi}} - Z^{+I}\frac{\partial}{\partial \tilde{\xi}} - Z^{+I}\frac{\partial}{\partial \tilde{\xi}^{+I}}.
\end{aligned} \tag{6.19}
$$

Now, repeating the steps of [18], we can easily verify that the field equations of motion corresponding to (6.18):

$$p^2\tilde{\Phi}^{(\mathrm{red})}=0$$

together with the physical state conditions [30] (G is the ghost-number operator):

$$\tilde{Q}^{(\mathrm{red})}_{\mathrm{BRST}}\tilde{\Phi}^{(\mathrm{red})}=0\,,$$

$$G\tilde{\Phi}^{(\mathrm{red})}=0\,,$$

$$\tilde{\Phi}^{(\mathrm{red})}\sim\tilde{\Phi}^{(\mathrm{red})}+\tilde{Q}^{(\mathrm{red})}_{\mathrm{BRST}}\tilde{\Phi}'\qquad\text{for any }\tilde{\Phi}^*\,,$$

and accounting for the reality condition (6.16) yield the same Lorentz-covariant solution $\hat{\Phi}(z,u,v)$ (5.14) (5.15) as for the Dirac constrained equations (5.1)–(5.5). Here:

$$\hat{\Phi}\left(p,\phi^{+\frac{1}{2}k},u,v,w\right)=\tilde{\Phi}^{(\mathrm{red})}_0\left(p,\phi^{+\frac{1}{2}k},u,v,w,Y_{ab},Y^{-+},Y^{+a},Z^{IJ},Z^{+-},Z^{+I}\right),$$

$$\frac{\partial}{\partial Y_{ab}}\tilde{\Phi}^{(\mathrm{red})}_0=\cdots\frac{\partial}{\partial Z^{+I}}\tilde{\Phi}^{(\mathrm{red})}_0=0$$

and $\tilde{\Phi}^{(\mathrm{red})}_0$ is the zeroth order term in the ghost expansion of $\tilde{\Phi}^{(\mathrm{red})}$.

7. Conclusions

In the present paper we succeeded to reformulate the $D=10$, $N=1,2$ BS superparticles as constrained systems possessing Lorentz-covariant and functionally independent first class constraints only.

The key ingredient of our formalism was the introduction of two generations of additional (pure gauge) bosonic degrees of freedom – the harmonics corresponding to the homogenous spaces $\mathrm{SO}(1,9)/\mathrm{SO}(8)\times\mathrm{SO}(1,1)$ and $\mathrm{SO}(8)/\mathrm{SU}(4)\times\mathrm{U}(1)$.

The above harmonics are crucial in three contexts:

(a) in the Lorentz-covariant separation of the fermionic constraints into functionally independent first class and second class parts;

(b) in converting the 8N covariant second class constraints into 4N covariant first class constraints,

(c) in proving the physical equivalence of the covariant harmonic superparticle with the standard BS superparticle treated in the light-cone formalism.

We also succeeded to find a covariant unconstrained superfield action of the $D=10$ linearized SYM theory.

In order to construct the complete nonlinear action within the BRST approach [18] one would have to find simultaneously both higher nonlinear interacting terms in the BRST action (6.6) as well as the higher nonlinear terms in the BRST transformation law (6.10) in such a way that the complete action remains BRST-invariant. This task, although technically involved, is tractable since one expects finite number of higher nonlinear terms (unlike the case of $D=10$, $N=2$ supergravity).

There exists another harmonic superspace formalism [16] outside the BFV-BRST approach which offers a way to directly construct the complete nonlinear unconstrained superfield action for $D=10$ SYM theory.

Our conjecture is that the latter result will be achieved after introducing a third generation of harmonics besides the two generations employed in the present formalism. This work is now in progress.

A very important open problem is whether the present mechanism of converting the $8N$ second class constraints into an equivalent set of $4N$ constraints extends to the case of the GS superstring; thus avoiding the necessity of covariant [21] gauge fixing of the fermionic κ- and reparametrization invariances.

We are grateful for the warm hospitality and the stimulating atmosphere of the CERN Theory Division where this work was initiated. Two of us (E.N. and S.P.) are deeply indebted to E. Sokatchev and S. Kalitzin for numerous illuminating discussions on the fundamentals of harmonic superspace. It is a pleasure for E.N. and S.P. to thank also the Weizmann Institute of Science, Rehovot, for most cordial hospitality. One of us (S.S.) would like to thank J-W van Holten and Y. Eisenberg for very instructive discussions. We all are indebted to A. Schwimmer and N. Seiberg for their incentive remarks.

Appendix

$D=10$, $D=8$ AND $D=6$ SPINOR CONVENTIONS

The $D=10$ γ-matrices and $D=10$ charge conjugation matrix are taken in the following representation:

$$\Gamma^{\mu}=\begin{pmatrix} 0 & (\sigma^{\mu})_{\alpha}^{\dot\beta} \\ (\tilde\sigma^{\mu})_{\dot\alpha}^{\beta} & 0 \end{pmatrix},$$

$$C_{10}=\begin{pmatrix} 0 & C^{\alpha\dot\beta} \\ (-C)^{\dot\alpha\beta} & 0 \end{pmatrix},$$

$$\Gamma^{11}\equiv\Gamma^{0}\Gamma^{1}\dots\Gamma^{9}=\begin{pmatrix} \delta_{\alpha}^{\beta} & 0 \\ 0 & -\delta_{\dot\alpha}^{\dot\beta} \end{pmatrix}.$$

Indices of $D=10$ left- (right-) handed MW spinors ϕ_α, $\psi_{\dot\alpha}$ are raised by means of C_{10}:

$$\phi^{\dot\alpha} = (-C)^{\dot\alpha\beta}\phi_\beta ,$$

$$\psi^{\alpha} = C^{\alpha\dot\beta}\psi_{\dot\beta} .$$

Throughout the paper we use $D=10$ σ-matrices with undotted indices only:

$$(\sigma^\mu)^{\alpha\beta} = C^{\alpha\dot\alpha}(\tilde\sigma^\mu)_{\dot\alpha}^{\ \beta},$$

$$(\sigma^\mu)_{\alpha\beta} = (-C)^{-1}_{\beta\dot\beta}(\sigma^\mu)_\alpha^{\ \dot\beta},$$

$$(\sigma^\mu)_{\alpha\gamma}(\sigma^\nu)^{\gamma\beta} + (\sigma^\nu)_{\alpha\gamma}(\sigma^\mu)^{\gamma\beta} = -2\delta_\alpha^\beta\eta^{\mu\nu},$$

$$\eta_{\mu\nu} = \mathrm{diag}(-,+,\dots,+) .$$

For the $D=8$ γ-matrices and $D=8$ charge conjugation matrix we use the following representation:

$$\Gamma_8^i = \begin{pmatrix} 0 & (\gamma^i)_a^{\ \dot b} \\ (\tilde\gamma^i)_{\dot a}^{\ b} & 0 \end{pmatrix},$$

$$C_8 = \begin{pmatrix} C^{ab} & 0 \\ 0 & (-C)^{\dot a\dot b} \end{pmatrix},$$

$$C^{ab} = C^{ba} .$$

Indices of SO(8) (s) and (c) spinors ϕ_α, $\psi_{\dot a}$ are raised as:

$$\phi^a = C^{ab}\phi_b , \qquad \psi^{\dot a} = (-C)^{\dot a\dot b}\psi_{\dot b} .$$

The $D=6$ γ-matrices and $D=6$ charge conjugation matrix are taken in the representation:

$$\Gamma_6^I = \begin{pmatrix} 0 & (\rho^I)_k^{\ \dot l} \\ (\tilde\rho^I)_{\dot k}^{\ l} & 0 \end{pmatrix},$$

$$C_6 = \begin{pmatrix} 0 & C^{k\dot l} \\ (C)^{\dot k l} & 0 \end{pmatrix},$$

$$C^{\dot k l} = C^{l\dot k} .$$

Indices of SO(6), i.e. SU(4), (4) and $(\bar{4})$ spinors ϕ^k, $\bar{\psi}^k$ are lowered by means of C_6^{-1}:

$$\phi_k = C_{kl}\phi^l,$$

$$\bar{\psi}_k = C_{kl}\bar{\phi}^l.$$

In particular, $D=6$ σ-matrices with undotted indices are antisymmetric:

$$\rho^I_{kl} = \left(\rho^I\right)^i_k C_{il} = -\rho^I_{lk}.$$

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